Optics

(1) Doppler Shift: $v = \lambda f$ sound: $f_r = \frac{f_s}{1 - v/v_s}$
light: $\Delta\lambda = 2v\gamma\lambda_0/c$

(2) Optics:

thin lens formula $\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$

focal length (lens) $1/f = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

(mirror) $f = R/2$

total internal refl.: $\sin\theta_c = n_1/n_2$

Rayleigh criterion: $\sin\theta_d = 1.22\ \lambda/D$

magnification: $M = -s_2/s_1 = -f_2/f_1$

(3) Diffraction

at a slit: $I = I_0\left(\frac{\sin(ak_o)}{ak_o}\right)^2\left(\frac{\sin(Ndk_o)}{\sin(dk_o)}\right)^2$

$k_o = (\pi/\lambda)\sin\theta = (k/2)\sin\theta$

a = thickness, d = distance between slits, N = # slits

max: $m\lambda = d\sin\theta$

(4) Misc.:

Polarizer halves Intensity

$I = |E|^2$

Planck length: $\ell = \sqrt{G\hbar/c^3}$

Thermal

(1) Energy

partition function: $Z = \sum e^{-E/kT}$

total internal energy: $U = \frac{NkT^2}{Z}\left(dZ/dT\right) = \frac{f}{2}NkT$

degrees of freedom: monatomic = 3

diatomic = 5

high temp = 7

$\langle E\rangle = \left(\sum E_n e^{-E_n/kT}\right)/Z$

Statistical

(1) Distributions

Maxwell-Boltzmann $N = g e^{-e/kT}$

Fermi-Dirak $N_n = N_0 / (1 + e^{-E_n/kT})$

(2) Rules

- free electrons (Fermi) obey Pauli
- model of the atom depends on T
- Wein's Law : $\lambda = b/T$
- approximation : $e^x = 1 + x \quad x << 1$
- fermi velocity : $V_F = \sqrt{2kT_F/m} \sim 10^6 \, m/s$
- Debye + Einstein : 3N Oscillators
- air molecule : $r \sim 2 \times 10^{-10} m$, $\eta = 5 \times 10^{25} \, m^{-3}$
- pressure : $P_F > P_C > P_B$

Quantum

(1) BlackBody Spectrum

$E(\nu, t) = \left(\frac{8\pi h}{c^3}\right) \nu^3 / (e^{h\nu/kT} - 1)$

photoelectric : $E_p = h\nu - W$

compton : $\lambda' - \lambda = (h/m_e c)(1 - \cos\Theta)$

de Broglie : $\lambda = h/p$

(2) Bohr Model

radii : $mvr = n\hbar$

spectra : $1/\lambda = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

energy : $E_n = (Z^2 e^4/n^2) E_1 \quad E_1 = 13.6 eV$

Atomic

(1) orbital stuff

orbital: $\vec{l} = \vec{n} \times \vec{p}$

$L = \Sigma l_i$

spin: $\vec{s}$

ang momentum: $j = l + s$

(2) Angular momentum:

$[L_x, L_y] = L_z(ik)$

$[L_x, L_z^2] = 0$

$L^2 |l, m\rangle = \hbar^2 l(l+1) | \rangle$

$L_z |l, m\rangle = \hbar m | \rangle$

(3) Spin:

$S = 1/2$ particles:

$S_i = \frac{1}{2} \hbar \sigma_i$

pauli matrices:

$\sigma_x = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

(4) Quantum #:

n = quantum #	$n = 1, 2, 3$
l = orbital	$l = 0, 1, \ldots n-1$
m_l = magnetic quantum	$m_l = -l, \ldots +l$

shell structure:

n	# electrons	notation
1	2	K
2	8	L
3	18	M
4	32	N

Mechanics

(1) Kinematics

$x(t) = x_0 + v_0 t + \frac{1}{2} a t^2$ $\quad v(t) = v_0 + at$

$p = mv \quad F = \dot{p}$

$F_{ext} = 0 \Rightarrow p$ conserved

(2) Newton's Laws

(1) w/o force, constant v $\quad$ (2) $F = ma$ $\quad$ (3) $F_{12} = -F_{21}$

(3) Work + Energy

$W = F \cdot d = \Delta K = -U$ $\quad F = -\nabla U$

$K = \frac{1}{2} m v^2 = p^2/2m$ $\quad U_{grav} = GmM/r \; \hat{r}$

(4) Oscillations

$F = -kx \quad U_x = \frac{1}{2} k x^2$

SHM: $m\ddot{x} = -kx \quad \omega = \sqrt{k/m} \quad T = 2\pi/\omega$

Solutions: $x = C_1 e^{i\omega t} + C_2 e^{-i\omega t}$

$x = x_0 \cos(\omega t) + v_0/\omega \, (\sin \omega t)$

$\omega_{small} = \sqrt{mgr/I}$

damped: $m\ddot{x} + b\dot{x} + kx = 0$

Fourier Series: $x(t) = \sum_0 A_n \cos(n\omega t - \delta_n)$

(5) Rotation about a Fixed Axis

$\vec{v} = \vec{\omega} \times \vec{r}$

$\vec{F}_{cor} = 2m\vec{\omega} \times \vec{v}$

ang. momentum (conserved) $L = I\omega = \vec{r} \times m\vec{v}$

moment of inertia: $I = I_{cm} + mR^2$

$I_{rod,end} = \frac{1}{3} MR^2 \quad I_{rod,center} = \frac{1}{12} mR^2$

$I_{disk,center} = \frac{1}{2} mR^2$

torque: $\vec{\tau} = \dot{L} = I\alpha = \vec{r} \times \vec{F}$

$F_{cent} = mv^2/r = m\omega r$

E+M

Maxwell's EQ's:

I. $\nabla \cdot E = \rho / \epsilon_0$ $\quad \epsilon > \epsilon_0 \quad D = \epsilon E \quad \nabla \cdot D = \rho$

II. $\nabla \cdot B = 0$

III. $\nabla \times E = -\partial B / \partial t$

IV. $\nabla \times B = \mu_0 J + \mu_0 \epsilon_0 \partial E / \partial t$ $\quad \mu_0 \epsilon_0 = 1/c^2$

Boundary Conditions (Interface)

I. $E_1^\perp - E_2^\perp = \sigma_f$

II. $E_1^\parallel - E_2^\parallel = 0$

III. $B_1^\perp - B_2^\perp = K_f$

IV. $B_1^\parallel - B_2^\parallel = 0$

E+M Fields

General: $E = \int \rho \, dV / r^2$ $\quad B = \int \frac{\mu_0}{4\pi} \frac{I \, d\vec{l} \times \hat{r}}{r^2}$

specific: $E_{loop} = \frac{K q r^2}{(r^2 + z^2)^{3/2}}$ $\quad B_{loop} = \frac{\mu_0 I}{2} \frac{r^2}{(r^2+z^2)^{3/2}}$

$E_{sphere}^{out} = \frac{K Q_{enc}}{r^2}$

$B_{center} = \mu_0 I / 2r$

$E_{wire} = \frac{2\lambda}{r}$

$B_{wire} = \mu_0 I / 2\pi R$

$E_{sphere}^{in} = \frac{\rho r}{3}$

$E_{plane} = 2\pi\sigma$ $\quad B_{plane} = \mu_0 K / 2$

$E_{ppc} = \frac{2\sigma}{\epsilon_0}$ $\quad B_{solonoid} = \frac{4\pi I N}{L}$

Forces: $F_e = qE$ $\quad F_B = q\vec{v} \times \vec{B}$

$F_{wire} = I \vec{l} \times \vec{B}$ $\quad J = \frac{I}{A} = n e v_{drift}$

E+M Waves

$$\vec{S} = \frac{1}{\mu_o}(\vec{E} \times \vec{B})$$

wave equation $\frac{1}{c^2}\frac{d^2\varphi}{dt^2} = \frac{\partial^2\varphi}{\partial x^2}$

group velocity: $d\omega/dk$
phase velocity: ω/k

$$\nu = \frac{1}{T} = \frac{kv}{2\pi} = \frac{v}{\lambda} = \frac{\omega}{2\pi}$$